A New Method for Compositing Reaction History Data

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One of the most useful diagnostics for nuclear tests is the reaction history curve, which describes the evolution of the criticality in time. The reaction history is inferred from oscilloscope data, recorded in the form of a so-called Rossi trace. A previous paper [1] described methods for converting a Rossi trace into a time series. In this paper we consider the compositing problem: how do we combine the time series from different oscilloscopes?

In Fig. 1 we show four simulated time series in the log-signal, each from a different Rossi trace. The problem is to combine all of this data into a single continuous curve, and then estimate the derivative of the combined curve, which is the criticality $\alpha(t)$.

Although this problem has been around for 50 years, it has resisted a completely satisfactory solution. To address this problem, we have developed a new compositing method, which we call SS-ANCOVA, for "smoothing splines and analysis of covariance." It can be derived, using Bayesian inference, from a clearly stated set of reasonable assumptions. It is sufficiently flexible to combine all relevant sources of uncertainty. It provides closed-form expressions for the uncertainty, which can be evaluated easily and rapidly on a workstation.

Analysis of covariance is a statistical technique for combining data from different sources. ANCOVA models combine treatment effects, which are specific to each source, with a regression model, which is the same for all sources. In our case, the "treatment" is the measurement of the data by an individual scope, and the treatment variables are the parameters characterizing the individual scope. Typically ANCOVA is used to improve estimates of the treatment variables by accounting for the shared dependence on regression parameters. In our application, ANCOVA is used to obtain estimates of the regression parameters by integrating out the treatment variables. In SS-ANCOVA, we use smoothing splines for our regression model.

We illustrate this formalism with the data from Fig. 1. The four time series have been derived from a single continuous signal, but they have been displaced vertically by different amounts so as to occupy a common range. The displacements correspond to different sensitivities on different scopes. If we knew the displacements, we could reconstruct a single smooth curve.

In applying SS-ANCOVA to this problem, we take our "treatment variables" to be the displacements β_i. Our regression model is $f_\gamma(t)$, where γ parameterizes a space of splines. Our data model is then

$$y_{ij} = \beta_i + f_\gamma(t_{ij}) + \varepsilon_{ij}$$

where ε_{ij} is measurement error. We compute the joint posterior distribution of β and γ, taking as "prior information" the requirement that the curve be smooth. To obtain the posterior curve distribution, we integrate over β.

In Fig. 2, we show the estimate of $f'(t)$ provided by our algorithm. The solid black line is the estimate, and the dotted line is the true value. The grey lines are 95% Bayesian uncertainty bands.

The estimate is intuitively reasonable. The true value, $f'(t) = 1$, lies comfortably within the uncertainty bands. On the bottom we have plotted the domains of the four time series, using the same color coding as in Fig. 1. Note the large increase in uncertainty where the data are absent, and the reduction of uncertainty where the data overlap.

In general, oscilloscope models will be more complicated than those of the example, and will include distortion and calibration parameters, as well as prior information. Indeed, the essential difficulty in this problem is that many sources of uncertainty and information must be combined and properly weighted. It seems to be very difficult to do this without a coherent quantitative framework that is capable of embracing all relevant sources of uncertainty. SS-ANCOVA has been developed to provide such a framework.

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[1] T.C. Wallstrom, "The Use of Smoothing Splines to Assess Uncertainties in Alpha Curves," *T-Division Nuclear Weapons Program Highlights 2004–2005*, Los Alamos National Laboratory report LA-UR-05-3853 (June 2005).

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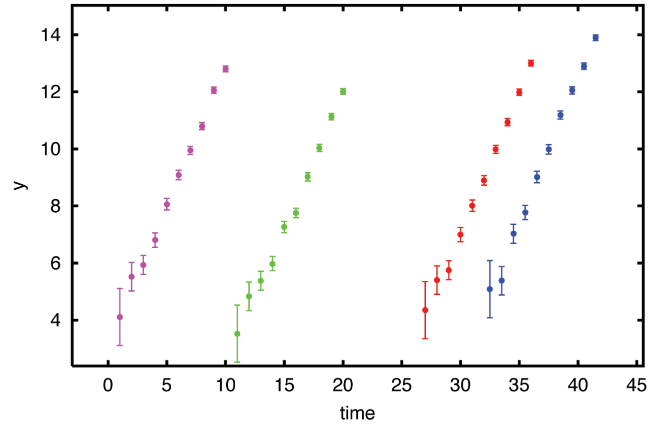

Fig. 1.
Four simulated time series.

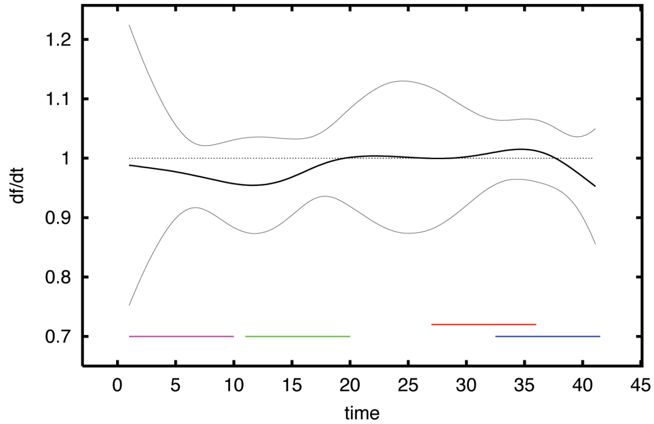

Fig. 2.
SS-ANCOVA estimate of f′(t), with uncertainty bands.